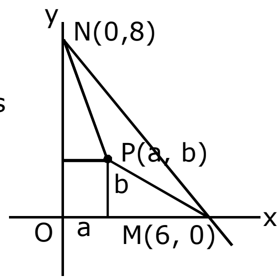
EXERCISE – III HINTS & SOLUTIONS

Sol.1 **(a)** Area of $\Delta POM = \Delta PON = \Delta PMN$

$= \frac{1}{3}\,\Delta OMN$



$= \frac{1}{3}\,\frac{1}{2}.\,6.\,8 = 8$sq. units

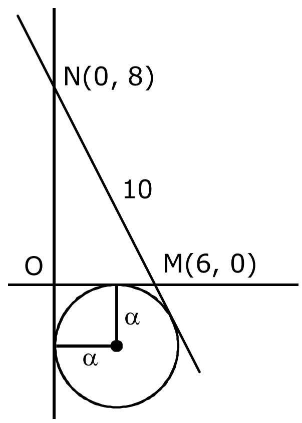
Now $\frac{1}{2}\,.\,6 \times b = 8$

$\Rightarrow b = \frac{8}{3}$

& $\frac{1}{2}\,8 \times a = 8 \Rightarrow a = 2 \Rightarrow (2, \frac{8}{3})$

(b) $\alpha = \frac{-6(0)+8(6)+10(0)}{-6+8+10} = \frac{48}{12}$

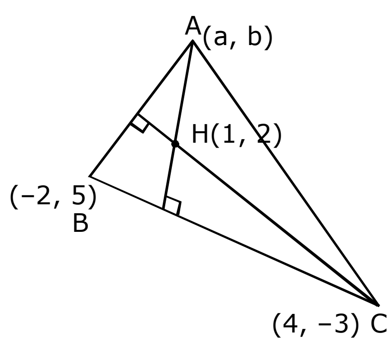
$\alpha = 4$



Sol.2 Let third vertex is (a, b)

$m_{AH}\, m_{BH} = -1$

$\frac{b-2}{a-1}\,.\,\frac{8}{-6} = -1$



$\Rightarrow 3a - 4b + 5 = 0$ ....(i)

& $m_{AB}\,.\,m_{HC} = -1$

$\frac{b-c}{a+2}\,.\,\frac{5}{-3} = -1$

$\Rightarrow 3a - 5b + 31 = 0$ ....(ii)

solve (i) & (ii) (a, b) $\Rightarrow$ (33, 26)

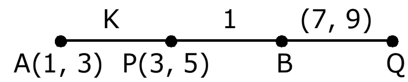
Sol.3 Since A $\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$

$\Rightarrow \text{ar (ABC)} = \frac{1}{2}\left|\left(\frac{5k+1}{k+1} - 5\left(\frac{3k-5}{k+1}\right)\right) + \left(-2\left(\frac{3k-5}{k+1}\right) - 7\left(\frac{5k+1}{k+1}\right) + 37\right)\right|$

$\Rightarrow 4 = \left|\frac{-14k+66}{2(k+1)}\right| \quad \Rightarrow \frac{-14k+66}{2(k+1)} = \pm 4$

$\Rightarrow k = 7$ or $31/9$

Sol.4 Internally



$\frac{7k+1}{k+1} = 3,\ \frac{9k+3}{k+1} = 5$

$4k = 2 \qquad \Rightarrow k = \frac{1}{2}$

P divide internally AB in 1 : 2

$\Rightarrow$ So Q. divide externally AB in 1 : 2

Q . $\left(\frac{7-2}{1-2}, \frac{9-6}{1-2}\right) = (-5, -3)$

Sol.5 $L_1 : 5x-y-4=0$

$L_2 : 3x+4y-4=0$

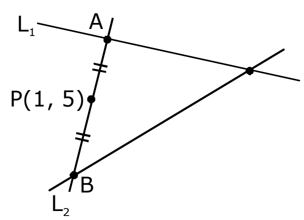
Line AB

$y - 5 = m\,(x-1)$

$mx - y + 5 - m = 0$

$5x - y - 4 = 0$

$- \quad + \quad +$



for A x = $\left(\frac{m-9}{m-5}\right)$ For B x = $\left(\frac{4m-16}{4m+3}\right)$

AP = BP

$\frac{1}{2}\left[\frac{m-9}{m-5} + \frac{4m-16}{4m+3}\right] = 1$

$4m^2 - 33m - 27 + 4m^2 - 46m + 80$

$= 8m^2 - 34m - 30$

$m = \frac{83}{35}$

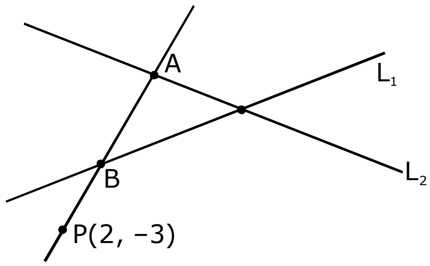
$y - 5 = \frac{83}{35}(x-1)$

$\Rightarrow 35y - 175 = 83x - 83 \Rightarrow 83x - 35y + 92 = 0$

Sol.6 $\frac{PA}{PB} = \frac{3}{2} \Rightarrow PA = 3r,\ PB = 2r$

$L_1 : x - 2y + 7 = 0$

$L_2 : x + 3y - 3 = 0$



Parametric equatoin of PA

$$\frac{x-2}{\cos\theta} = \frac{y+3}{\sin\theta}$$

coordinate A $(2 + 3r\cos\theta, -3 + 3r\sin\theta)$ satisfy L_1

coordinate B $(2 + 2r\cos\theta, -3 + 2r\sin\theta)$ satisfy L_2

$2 + 3r\cos\theta + 6 - 6r\sin\theta + 7 = 0$

$r\cos\theta - 2r\sin\theta + 5 = 0$ ....(i)

& $2 + 24\cos\theta - 9 + 6r\sin\theta - 3 = 0$

$r\cos\theta + 3r\sin\theta - 5 = 0$ ....(ii)

solve (i) & (ii) $r\cos\theta = -1$, $r\sin\theta = 2$

$\Rightarrow \tan\theta = -2$

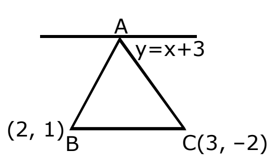
equation of AB is $y + 3 = -2(x - 2)$

$2x + y - 1 = 0$

Sol.7 Let vertex A $(a, a + 3)$

$\Delta ABC = 5$ sq. units

$$\frac{1}{2}\begin{vmatrix} a & a+3 & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{vmatrix} = \pm 5$$



$\Rightarrow (3)a - (a + 3)(-1) + (-4 - 3) = \pm 10$

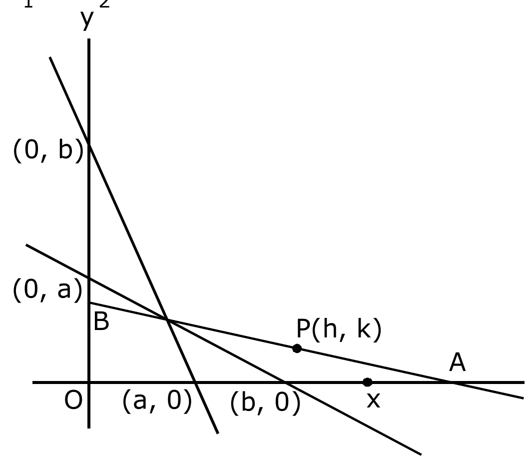
$\Rightarrow 4a = \pm 10 + 4 \quad \Rightarrow a = \frac{7}{2}, \frac{-3}{2}$

A $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

Sol.8 $L_1 : \frac{x}{b} + \frac{y}{a} = 1$

$L_2 : \frac{x}{b} + \frac{y}{a} = 1$

$L_1 + \lambda L_2 = 0$



$$\left(\frac{x}{a} + \frac{y}{b} - 1\right) + \lambda\left(\frac{x}{a} + \frac{y}{b} - 1\right) = 0$$

$$\left(\frac{1}{a} + \frac{\lambda}{b}\right)x + \left(\frac{1}{b} + \frac{\lambda}{a}\right)y = \lambda + 1$$

$$A\left(\frac{\lambda+1}{\frac{1}{a}+\frac{\lambda}{b}}, 0\right),\ B\left(0, \frac{\lambda+1}{\frac{1}{b}+\frac{\lambda}{a}}\right)$$

Mid point of AB is P(h, k)

$$2h = \frac{\lambda+1}{\frac{1}{a}+\frac{\lambda}{b}} \ \&\ 2k = \frac{\lambda+1}{\frac{1}{b}+\frac{\lambda}{a}}$$

$$\lambda = \frac{1-\frac{2h}{a}}{\frac{2h}{b}-1} = \frac{1-\frac{2k}{b}}{\frac{2k}{a}-1}$$

$$\Rightarrow \frac{(a-2h)(2k-a)}{a^2} = \frac{(b-2k)(2h-b)}{b^2}$$

$\Rightarrow b^2(2ak - a^2 - 4hk + 2ah)$

$= a^2(2bh - 4hk + 2bk)$

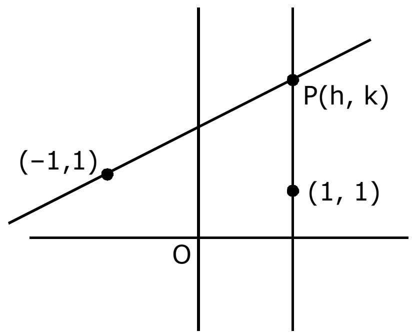
$\Rightarrow 2(a^2 - b^2)hk = (h + k)ab(a - b)$

$\Rightarrow 2(a + b)hk = (h + k)ab$ locus of (h, k)

$\Rightarrow 2xy(a + b) = ab(x + y)$

Sol.9 $|m_1 - m_2| = 2$

$$m_1 = \frac{k-1}{h-1},\ m_2 = \frac{k-1}{h+1}$$

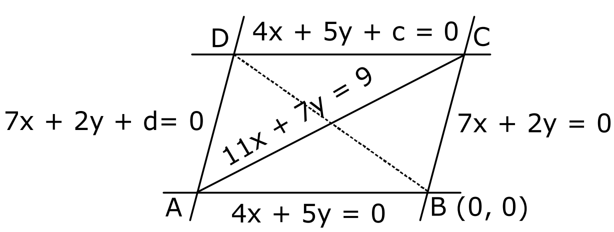


$$\Rightarrow \left(\frac{k-1}{h-1} - \frac{k-1}{h+1}\right)^2 = 4$$

$$\Rightarrow (k-1)^2 \left(\frac{2}{h^2-1}\right)^2 = 4$$

$\Rightarrow (k-1)^2 = (h^2-1)^2$
$\Rightarrow (y-1) = \pm (x^2-1)$
$\Rightarrow y = x^2$ or $y = 2 - x^2$

Sol.10 B should be (0, 0)
given diagonal AC is



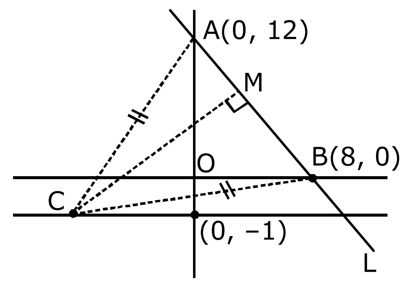
$11x + 7y = 9$ ....(i)
equation of AC
$(4x + 5y + C)(7x + 2y + d)$
$\quad - (4x + 5y)(7x + 2y) = 0$
$(7C + 4d)x + (2C + 5d)y + cd = 0$ ...(ii)
compair (i) & (ii)

$$\underbrace{\frac{7c+4d}{11} = \frac{2c+5d}{7}} = \frac{cd}{-9}$$

$$49c + 28d = 22c + 55d \quad \left\{ \begin{array}{l} \dfrac{7c+4d}{11} = \dfrac{cd}{-9} \\ \Rightarrow\ 9c + C^2 = 0 \\ C(C+9) = 0 \end{array} \right.$$
$\Rightarrow\ c = d$

C = 0 not possible
$\Rightarrow\ c = -9$ & $d = -9$
Diagonal BD is
$(4x + 5y)(7x + 2y - 9)$
$\quad - (4x + 5y - 9)(7x + 2y) = 0$
$\Rightarrow -9(4x + 5y) - (-9)(7x + 2y) = 0$
$\Rightarrow 3x - 3y = 0 \quad \Rightarrow x - y = 0$

Sol.11 L : $3x + 2y = 24$
ΔABC isoscles, (AC = BC)
M is mid point of AB
M (4, 6)



equation of $\perp$ bisector of AB

$$y - 6 = \frac{2}{3}(x-4) \Rightarrow 2x - 3y + 10 = 0$$

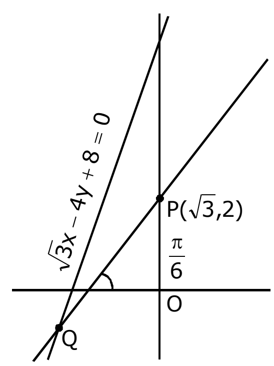
$$C\left(-\frac{13}{2}, -1\right),\ \Delta ABC = \frac{1}{2}(AB).(CM)$$

$$= \frac{1}{2}\sqrt{208}\cdot\frac{7}{2}\sqrt{13}$$

$$= \frac{7}{4} \times 4\sqrt{13}\sqrt{13} = 91 \text{ sq. units}$$

Sol.12 Let PQ = r
equation of PQ

$$\frac{x-\sqrt{3}}{\cos\frac{\pi}{6}} = \frac{y-2}{\sin\frac{\pi}{6}} = r$$



$$\Rightarrow Q\left(\sqrt{3} + \frac{\sqrt{3}r}{2}, 2 + \frac{r}{2}\right)$$

satisfy given line

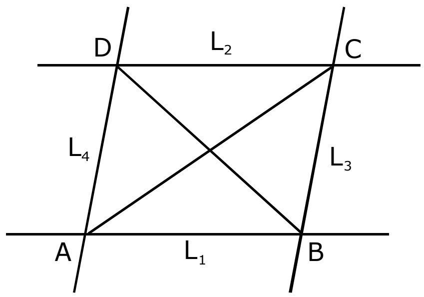
$$\Rightarrow \sqrt{3}\left(\sqrt{3} + \frac{\sqrt{3}r}{2}, 2 + \frac{r}{2}\right) + 8 = 0$$

$$\Rightarrow 3 + \frac{3}{2}r - 8 - 2r + 8 = 0 \Rightarrow \frac{r}{2} = 3$$

$\Rightarrow r = 6$

Sol.13 Diagonal AC

$L_1L_3 - L_2L_4 = 0$



$(ax + by + c)\,(a'x + b'y + c) - (ax+by+ c')$

$(a'x + b'y + c') = 0$

$\Rightarrow c(ax + by) + c\,(a'x + b'y)$

$-c'\,(ax +by) - c'\,(a'x + b'y) + c^2 - c'^2 = 0$

$\Rightarrow (c - c')\,(a + a')\,x + (c - c')\,(b + b')\,y$

$+ (c + c')\,(c-c') = 0$

$\Rightarrow (a + a')\,x + (b + b')\,y + (c + c') = 0$

Diagonal BD

$L_1L_4 - L_2L_3 = 0$

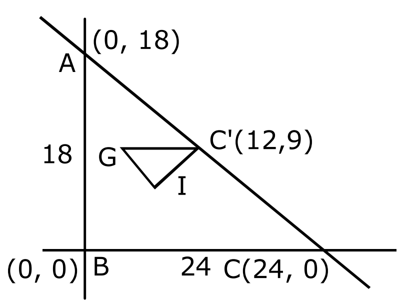
$(C-C')\,(a' - a)x + (c - c') + (b' - b)\,y = 0$

$\Rightarrow (a' - a)\,x + (b' - b)\,y = 0$

Sol.14 Sides are 18,m 24, 30

which are right triangle triplet

$(18^2 + 24^2 = 30^o)$



Let coordinates of A,B, C are A(0, 18), B(0, 0), C(24, 0) circumcentre c' (12, 9) centroid G(8, 6)

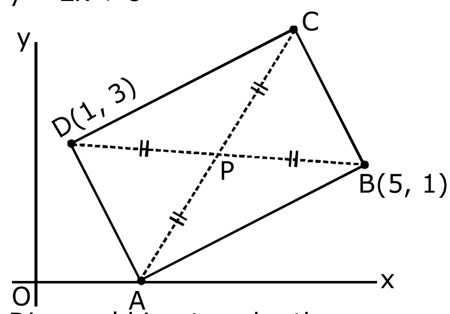
Incentre I $\left(\dfrac{0+0+18.24}{24+30+18}, \dfrac{24.18+0+0}{24+30+18}\right)$

$I \equiv (6, 6)$

area of $\Delta IC'G$ is = $\dfrac{1}{2}\begin{vmatrix} 6 & 6 & 1 \\ 12 & 9 & 1 \\ 8 & 6 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[18 - 24 + 0] = 3$ sq. units

Sol.15 $y = 2x + c$



Diagonal bisect each other

mid point of BD is P (3, 2)

y = 2x + C passing through P

$\Rightarrow 2 = 6 + c \qquad \Rightarrow c = -4$

$AP = BPO = CP = DP,\ BP = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

parametric form of AC

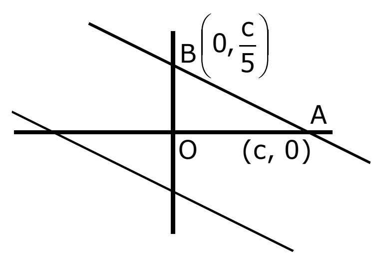
$\tan\theta = 2$, P(3, 2)

$$\frac{x-3}{\frac{1}{\sqrt{5}}} = \frac{y-2}{\frac{2}{\sqrt{5}}} = \pm\sqrt{5}$$

$x = 3 \pm 1,\ y = 2 \pm 2 \Rightarrow A(2, 0), C(4, 4)$

Sol.16 Line L can be

$x + 5y = C$



Area DOAB = 5 sq. units

$\left|\dfrac{1}{2}c \times \dfrac{c}{5}\right| = 5 \quad \Rightarrow |c^2| = 50 \Rightarrow c = \pm\sqrt{50}$

$\Rightarrow c = \pm 5\sqrt{2}$

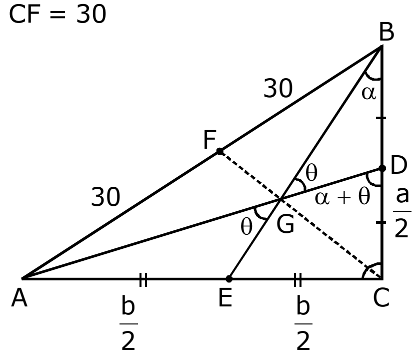
$L : x + 5y = \pm 5\sqrt{2}$

Sol.17 $L_{AD} \equiv y = x + 3$

$L_{BE} = y = 2x + 4$

AB = 60

CF = 30



$\tan\theta = \left|\frac{2-1}{1+2}\right| \Rightarrow \tan\theta = \frac{1}{3}$

& $a^2 + b^2 = 60^2$

In $\Delta BCE \Rightarrow \tan\alpha = \dfrac{\frac{b}{2}}{a} \Rightarrow \tan\alpha = \dfrac{b}{2a}$

In ΔACD $\tan(\theta + \alpha) = \dfrac{b}{\frac{a}{2}}$

$$\frac{\tan\theta + \tan\alpha}{1 - \tan\theta + \tan\alpha} = \frac{2b}{2a} \Rightarrow \frac{\frac{1}{3} + \frac{b}{2a}}{1 - \frac{b}{3.2a}} = \frac{2b}{a}$$

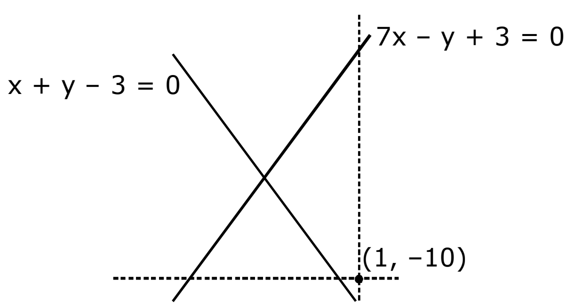
$\Rightarrow \dfrac{2a+3b}{6a-b} = \dfrac{2b}{a} \Rightarrow 2a^2 + 3ab = 12ab - 2b^2$

$\Rightarrow 9ab = 2(a^2 + b^2) \quad \Rightarrow 9ab = 2.60^2$

$\Rightarrow \frac{1}{2}ab = \dfrac{6a^2}{9} \Rightarrow \Delta ABC = \left(\dfrac{60}{3}\right)^2 = 20^2$

= 400 sq. units

Sol.18 One angle bisector is



$$\frac{x+y-3}{\sqrt{2}} = \frac{7x-y+3}{5\sqrt{2}}$$

$\Rightarrow 5x + 5y - 15 = 7x - y + 3$

$\Rightarrow 2x - 6y + 18 = 0 \Rightarrow x - 3y + 9 = 0$

Slope of angle bisectors are $\frac{1}{3}$ & – 3

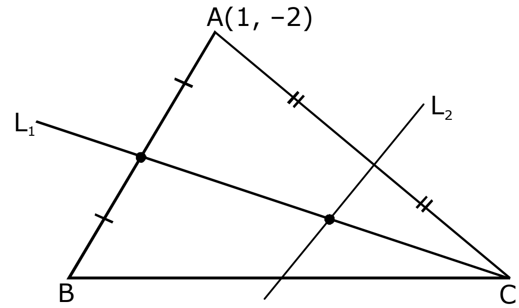
& req. lines are passing through (1, –10)

$(y + 10) = -3(x - 1)$ & $(y + 10) = \frac{1}{3}(x - 1)$

$3x + y + 7 = 0$ & $x - 3y = 31$

Sol.19 $L_1 \equiv x - y + 5 = 0$

$L_2 = x + 2y = 0$



Mirror image of A. w.r. to L_1 is B

$$\frac{x-1}{1} = \frac{y+2}{-1} = -\frac{(1+2+5)}{1^2 + (-1)^2}$$

$\Rightarrow x = -7, y = 6$

B (–7, 6)

& C is image ofr A w.r.t. to L_2

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{-2(-4)}{1^2 + 2^2}$$

$\Rightarrow x = \frac{11}{5}$ $y = \frac{-2}{5}$, C $\left(\frac{11}{5}, \frac{-2}{5}\right)$

Line BC is

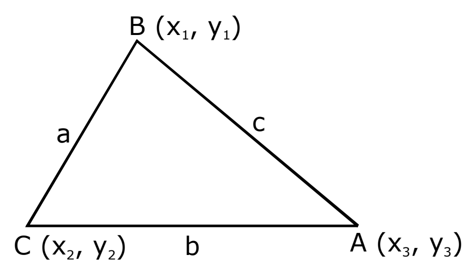
$y - 6 = -\frac{14}{23}(x + 7)$

$23y - 138 = -14x - 98 \Rightarrow 14x + 23y = 40$

Sol.20 $(x_1 - x_2)^2 + (y_1 - y_2)^2 = a^2$

$(x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2$

& $(x_3 - x_1)^2 + (y_3 - y_1)^2 = c^2$



$$\lambda \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$$

$= (a + b + c)(b + c - a)(c + a - b)(a + b - c)$

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $2s = a + b + c$

$\Rightarrow \lambda(2\Delta)^2 = 2s\,(2s - 2a)\,(2s - 2b)\,(2s - 2c)$

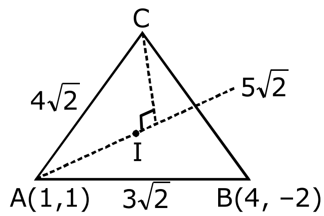
$\Rightarrow \lambda 2^2 \,.\, \Delta^2 = 2^4\, s(s - a)\,(s - b)\,(s - c)$

$\Rightarrow \lambda = 2^2 = 4$

Sol.21 AB = C = $3\sqrt{2}$

BC = a = $5\sqrt{2}$

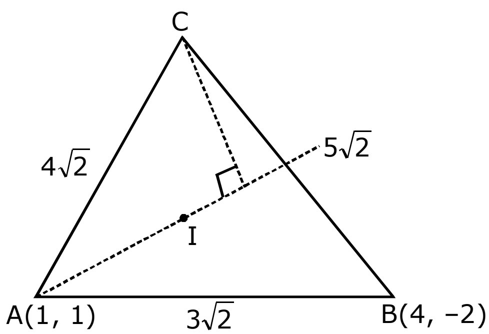
AC = b = $4\sqrt{2}$



Incentre

$$\left(\frac{5\sqrt{2}.1 + 4\sqrt{2}.4 + 5.3\sqrt{2}}{5\sqrt{2} + 4\sqrt{2} + 3\sqrt{2}}, \frac{5\sqrt{2}.1 + 4\sqrt{2} + 5.3\sqrt{2}}{5\sqrt{2} + 4\sqrt{2} + 3\sqrt{2}}\right)$$

$$\equiv \left(\frac{36\sqrt{2}}{12\sqrt{2}}, \frac{12\sqrt{2}}{12\sqrt{2}}\right) \Rightarrow \text{ I } (3, 1)$$



Incentre

$$\left(\frac{5\sqrt{2}.1 + 4\sqrt{2}.4 + 5.3\sqrt{2}}{5\sqrt{2} + 4\sqrt{2} + 3\sqrt{2}}, \frac{5\sqrt{2}.1 + 4\sqrt{2}(-2) + 5.3\sqrt{2}}{5\sqrt{2} + 4\sqrt{2} + 3\sqrt{2}}\right)$$

$$= \left(\frac{36\sqrt{2}}{12\sqrt{2}}, \frac{12\sqrt{2}}{12\sqrt{2}}\right) \Rightarrow \text{ I } (3, 1)$$

Interior angle angle bisector of A is

(y – 1) = 0 (x – 3) $\Rightarrow$ y = 1(|| to x-axis)

perpendicular from C on A I is x = 5

Sol.22 Area ΔABC = 70 sq. units

M $\left(\frac{35}{2}, \frac{39}{2}\right)$, sq. units

$$y - \frac{39}{2} = -5\left(x - \frac{35}{2}\right)$$

$\Rightarrow$ 10x + 2y = 214 satisfy (p, q)

$\Rightarrow$ 10p + 2q = 214 ....(i)

$$\text{Now} \begin{vmatrix} p & q & 1 \\ 12 & 12 & 1 \\ 23 & 20 & 1 \end{vmatrix} = 170$$

$\Rightarrow$ – p + 11q –197 = ± 140

$\Rightarrow$ – P + 11q - 337 ....(ii)

OR –P + 11Q = 57 ....(iii)

(i) & (ii)	or (i) & (iii)
P = 15, 32	p = 20, q = 7
A (15, 32)	A (20, 7)
p + q = 47	p + q = 27

max. (p + q) = 47

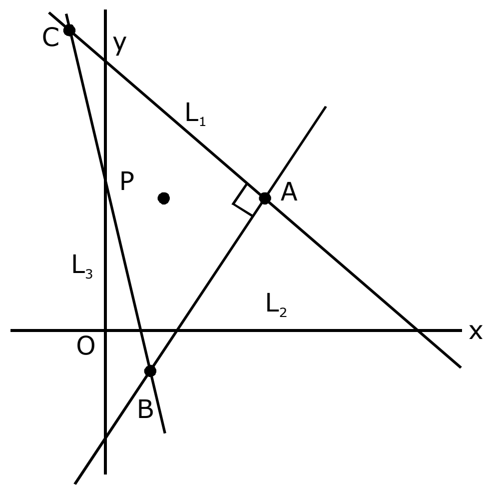
Sol.23 Origin lies outside

P (cos θ, sin θ)

L_1 : x + y – 2 = 0

L_2 : x – y – 1 = 0

L_3 : 6x + 2y – $\sqrt{10}$ = 0



If P lies inside ΔABC

L_1 (d) < 0 & L_2(2) < 0 & L_3(0) < 0

L_1 (P) < 0 & L_2(P) < 0 & L_3 (P) > 0

cosθ + sinθ –2 < 0

$$\sin\left(\theta + \frac{\pi}{4}\right) < \sqrt{2}$$

$\theta \in (0, \pi/4) \cup (3\pi/4, 4\pi)$ ...(i)

& cosθ – sinθ – 1 < 0

$$\cos\left(\theta + \frac{\pi}{4}\right) < \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{7\pi}{4}$$

$$\theta \in \left(0, \frac{3\pi}{2}\right) \text{ ....(ii)}$$

& 6 cosθ + 2 sinθ – $\sqrt{10}$ > 0

$$\frac{3}{\sqrt{10}}\cos\theta + \frac{1}{\sqrt{10}}\sin\theta > \frac{\sqrt{10}}{2\sqrt{10}}$$

$\sin(\theta + \alpha) > \frac{1}{2}$

$\frac{\pi}{6} < (\theta + \alpha) < \frac{5\pi}{6}$

$\theta \in [0, 2\pi]$ & $\theta \in \left(0, \frac{3\pi}{2}\right)$

& $\frac{\pi}{6} - \alpha < \theta < \frac{5\pi}{6} - \alpha$

$\tan\alpha = 3$

$\left(\frac{\pi}{6} - \alpha\right) < 0$ {$\theta > 0 \Rightarrow 0 \le \theta < \left(\frac{6\pi}{6} - \alpha\right)$

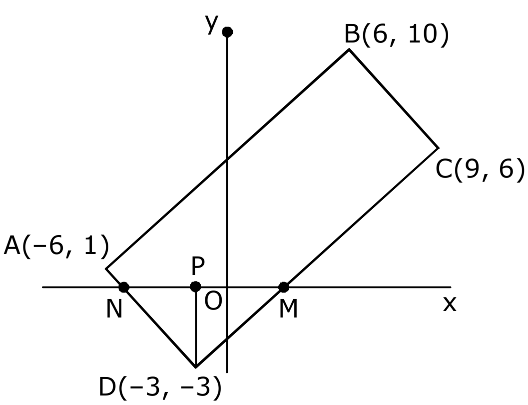
From (i) & (ii) & (iii)

$\theta \in \left(0, \frac{5\pi}{6} - \alpha\right) \Rightarrow 0 < \theta < \frac{5\pi}{6} - \tan^{-1} 3$

Sol.24 Area of ABCD = (AB) × (AD)

$AB = \sqrt{12^2 + 9^2} = 15$

$AD = \sqrt{3^2 + 4^2} = 5$



□ ABCD = 15 × 5 = 75 sq. units

Let N (a, 0)

$m_{AD} = m_{AN} \Rightarrow \frac{1+3}{-6+3} = \frac{0-1}{a+6}$

$\Rightarrow \frac{4}{-3} = -\frac{1}{a+6} \Rightarrow a = -\frac{21}{4}$

Let M (b, 0)

$m_{CD} = m_{DM} \Rightarrow \frac{6+3}{9+3} = \frac{0+3}{b+3} \Rightarrow \frac{3}{4} = \frac{3}{b+3}$

$\Rightarrow b = 1$

$MN = b - a = 1 + \frac{21}{4} = \frac{25}{4}$ & DP = 3

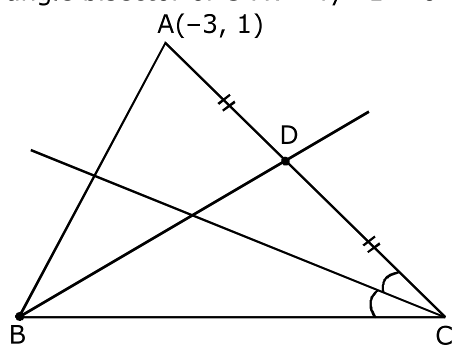
Area $\Delta DMN = \frac{1}{2} \times \frac{25}{4} \times 3 = \frac{75}{8}$ sq. units

area above x-axis $= 75 - \frac{75}{8}$

$= \frac{525}{8}$ sq.units $\Rightarrow 525 + 8 = 533$

Sol.25 B median $2x + y - 3 = 0$

angle bisector of C $7x - 4y - 1 = 0$



Let C on the line $7x - 4y - 100$

$C\left(\lambda, \frac{7\lambda + 1}{4}\right)$

D is mid point of AC lie median

$D\left(\frac{-3+\lambda}{2}, \frac{1 + \frac{7\lambda - 1}{4}}{2}\right)$

$2\left(\frac{-3+\lambda}{2}\right) + \frac{3+7\lambda}{8} - 3 = 0$

$-48 + 8\lambda + 3 + 7\lambda = 0 \Rightarrow \lambda = 3$

C (3, 5) & D(0, 3)

(C) line AC is $y - 3 = 0\frac{2}{3}(x - 0)$

$\Rightarrow 2x - 3y6 + 9 = 0$ (Q)

(P) will not a side Q (It's given median)

(A) Line AB A (–3, 1) satisfy (R) $4x + 7y + 5 = 0$

& (B) Line BC is only (S) $18x - y - 49 = 0$